# User Ordering and Subchannel Selection for Power Minimization in MIMO Broadcast Channels using BD-GMD

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# Overview

- Optimal Power Minimization
- Preliminaries: GMD and BD-GMD
- Subchannel Selection
- ZF-based Power Minimization
- Efficient Method
- Simulations
- Conclusion

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# Power Minimization

- Objective:
  - To minimize the transmit power for the MIMO broadcast channel,
  - given user rate requirements,
  - using Dirty Paper Coding.
- Cases:
  - Interference-Balancing (IB).
  - Zero-Forcing (ZF).

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# Optimal Power Minimization (1)

- Interference-Balancing (IB) Case
  - IUI, noise
  - Theoretical optimum – convex optimization
  - Better performance than ZF in low SNR region
  - Higher complexity than ZF case
  - Many iterations
  - Each iter. ↑ computations
  - No. iter. random

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# Optimal Power Minimization (2)

- Zero-Forcing (ZF) Case
  - Lower complexity
- User ordering and subchannel selection
  - Search over encoding orders and subchannel combinations
    - Limited predictable complexity
  - Suboptimal method with much reduced complexity
    - close to IB-optimal power

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# Preliminaries (1)

## Transmission Strategies for Single-User MIMO

- Singular Value Decomposition (SVD)
  - $\mathbf{H} = \mathbf{USV}^H$
  - Subchannel with different SNRs
- Geometric Mean Decomposition (**GMD**)[1]
  - $\mathbf{H} = \mathbf{PLQ}^H$
  - L is lower triangular, equal diagonal
  - Subchannels with identical SNRs



[1] Y. Jiang, J. Li and W. W. Hager, "Joint Transceiver Design for MIMO Communications Using Geometric Mean Decomposition,"
*IEEE Trans. Signal Processing*, vol. 53, no. 10, pp. 3791-3803, Oct. 2005.

# Preliminaries (2)

## Block-Diagonal GMD for Multi-User MIMO

$$H = P\,L\,Q^H$$

- $Q^H$: Unitary
- $P$: Block Diagonal & Unitary
- $L$: Lower Triangular

$$\begin{bmatrix} \mathbf{P}_1 & 0 & \dots & 0 \\ 0 & \mathbf{P}_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbf{P}_K \end{bmatrix} \qquad \begin{bmatrix} \mathbf{L}_1 & 0 & \dots & 0 \\ \mathrm{X} & \mathbf{L}_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{X} & \mathrm{X} & \dots & \mathbf{L}_K \end{bmatrix}$$

Each $P_i$ is unitary.

Each $L_i$ is equal diagonal.

**Block-equal-diagonal**

S. Lin, W. W. L. Ho, and Y.-C. Liang, "Block Diagonal Geometric Mean Decomposition (BD-GMD) for MIMO Broadcast Channels," *IEEE Trans. Wireless Commun.*, vol. 7, no. 7, pp. 2778-2789, Jul. 2008.

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# Subchannel Selection

- Block diagonal geometric mean decomposition
  with subchannel selection
  (BD-GMD-SS)
  - Successive GMD
  - Select singular values
  - Reduce transmit power

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# ZF-based Power Minimization (1)

- Rate req'm for each user = $R_k$ bps/Hz
  - SNR req'm
- # transmit antennas = $N_T$
- # receive antennas = $n_1, n_2, \ldots, n_K$
  sum=$N_R \leq N_T$
- Multiplexing: user k has $\eta_k$ data subchannels

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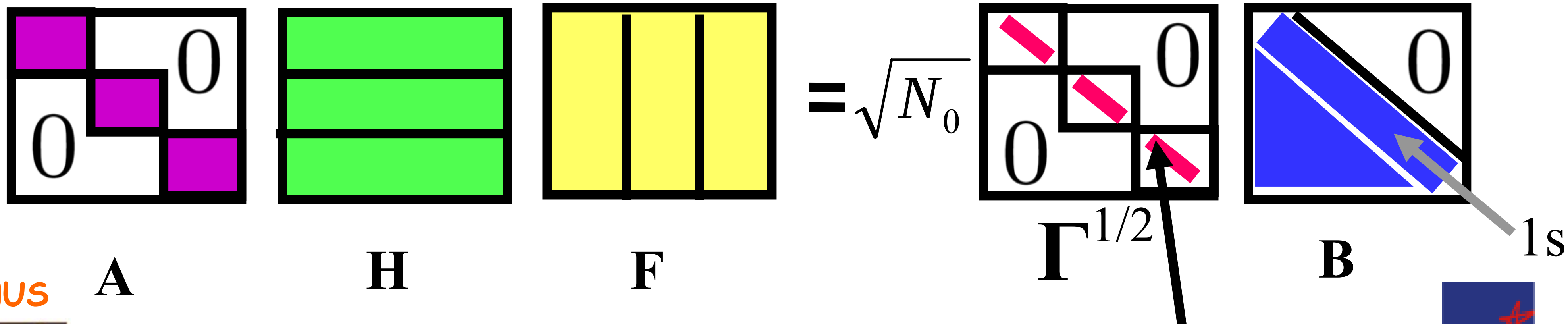
# ZF-based Power Minimization (2)

- Optimization problem:

$$
\begin{aligned}
&\text{minimize} && \mathrm{Tr}(\mathbf{F}^H \mathbf{F}) \\
&\text{subject to} && \mathbf{AHF} = \sqrt{N_0}\,\mathbf{\Gamma}^{1/2}\mathbf{B} \\
& && \mathbf{B} \in 9\ ,\ \mathbf{A} \in / \\
& && \|\mathbf{A}(i,:)\| = 1 \qquad \text{for} \quad 1 \le i \le N_D
\end{aligned}
$$




SNR req'm: $\mathbf{\Gamma}_k = \gamma_k \mathbf{I}_{\eta_k}$

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# ZF-based Power Minimization (3)

- Solution:
  - BD-GMD-SS: $\mathbf{P}^H\mathbf{H}\mathbf{Q} = \mathbf{L}$
  - let $\boldsymbol{\Lambda} = \mathrm{diag}(\mathbf{L})$
  - $$\boldsymbol{\Omega} = \sqrt{N_0}\boldsymbol{\Gamma}^{1/2}\boldsymbol{\Lambda}^{-1}, \quad \mathbf{F} = \mathbf{Q}\boldsymbol{\Omega},$$
    $$\mathbf{B} = \boldsymbol{\Omega}^{-1}\boldsymbol{\Lambda}^{-1}\mathbf{L}\boldsymbol{\Omega}, \quad \mathbf{A} = \mathbf{P}^H.$$
  - Minimum power = $E_s = \mathrm{Tr}(\mathbf{F}^H\mathbf{F}) = \mathrm{Tr}(\boldsymbol{\Omega}^2)$

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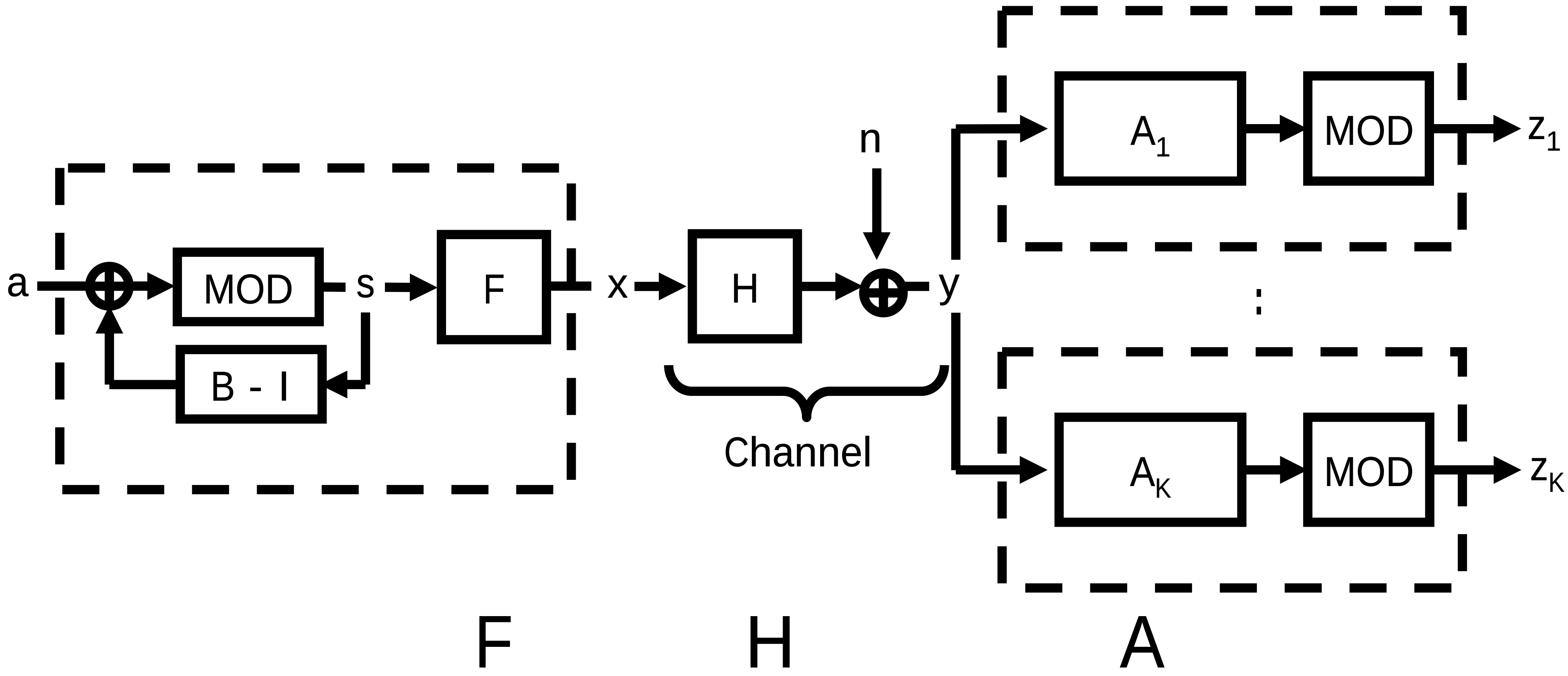
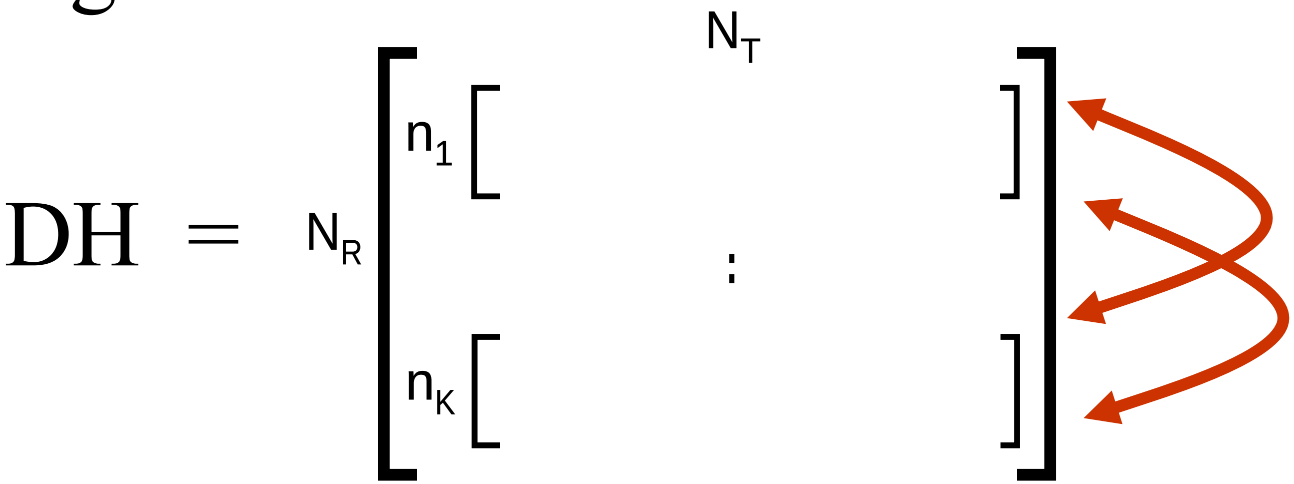
# Transceiver Design



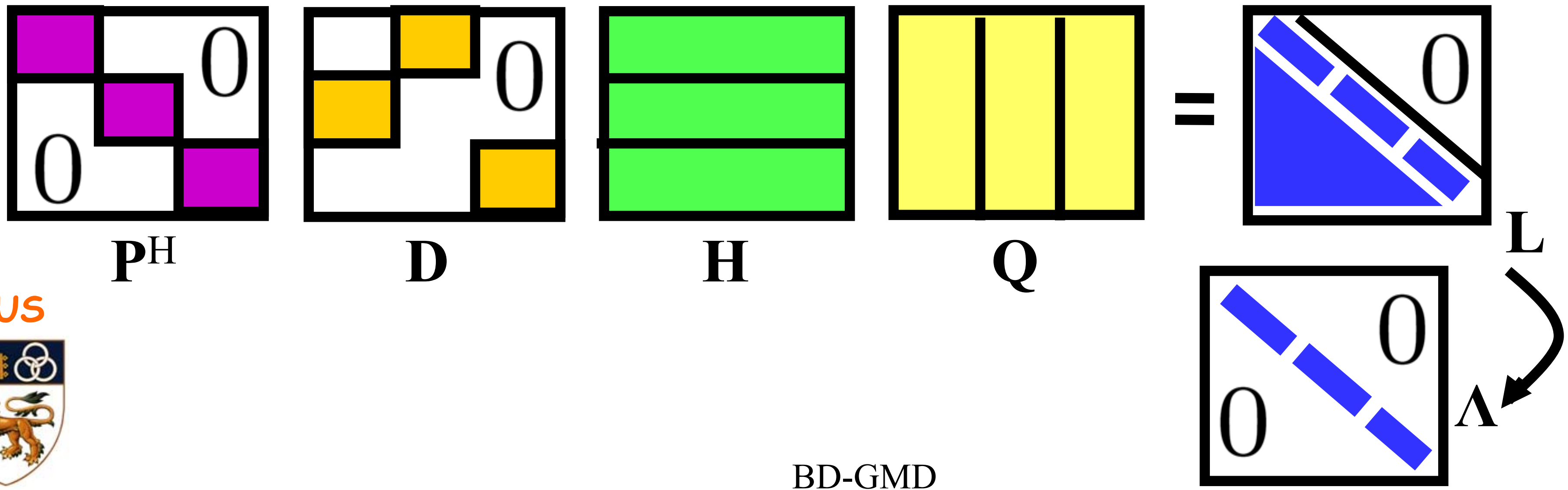

$$\mathbf{AHF} = \sqrt{N_0}\mathbf{\Gamma}^{1/2}\mathbf{B}$$

# User Ordering

- Rearranging the Channel Matrix




to achieve minimum transmit power.

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# Efficient Method

- Best Choice Ordering
  - Best of three methods
  - Successive selection of users
  - Top down manner
- Optimum subchannel selection

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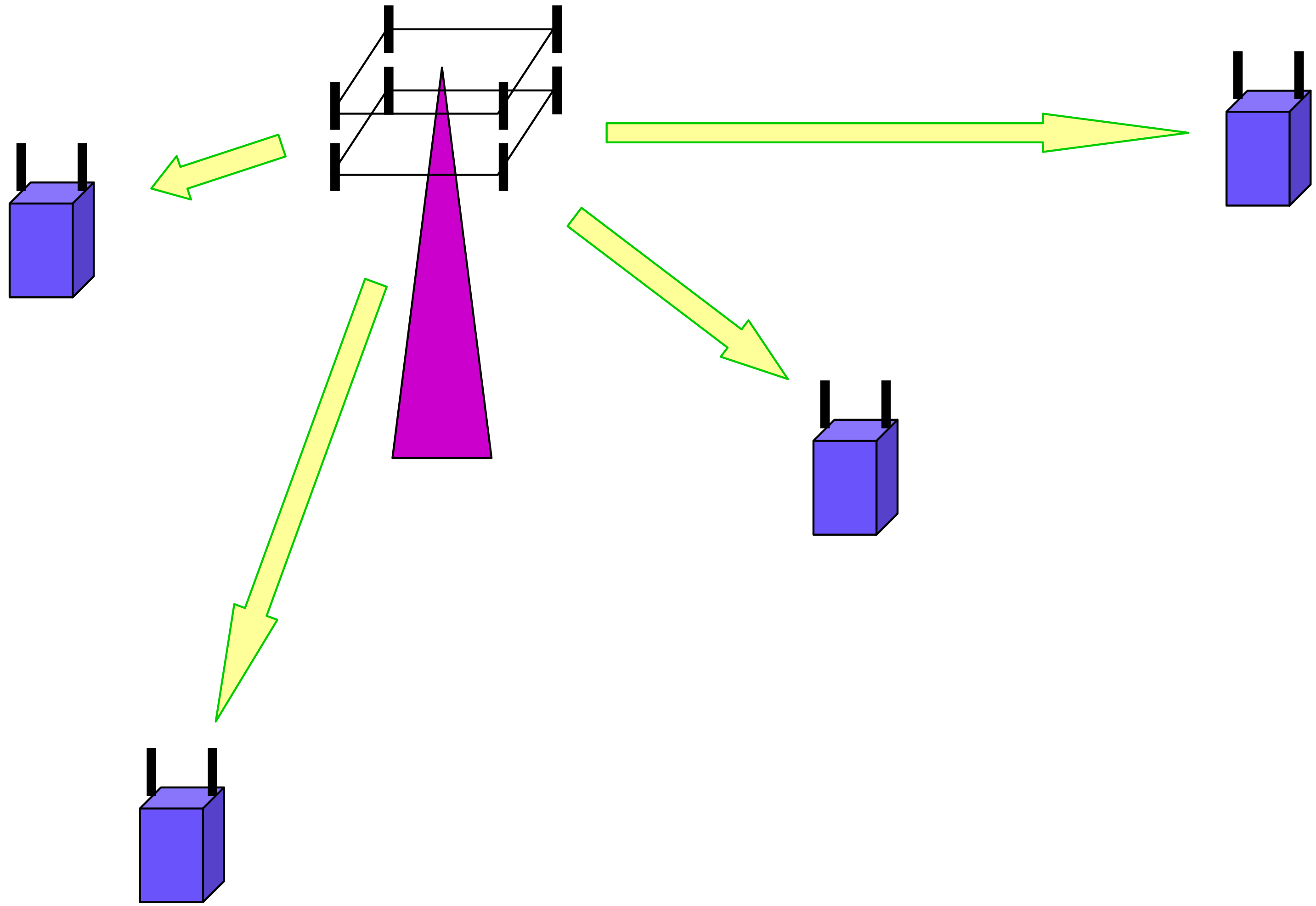
# Simulation Results



# Equal Rate Requirements

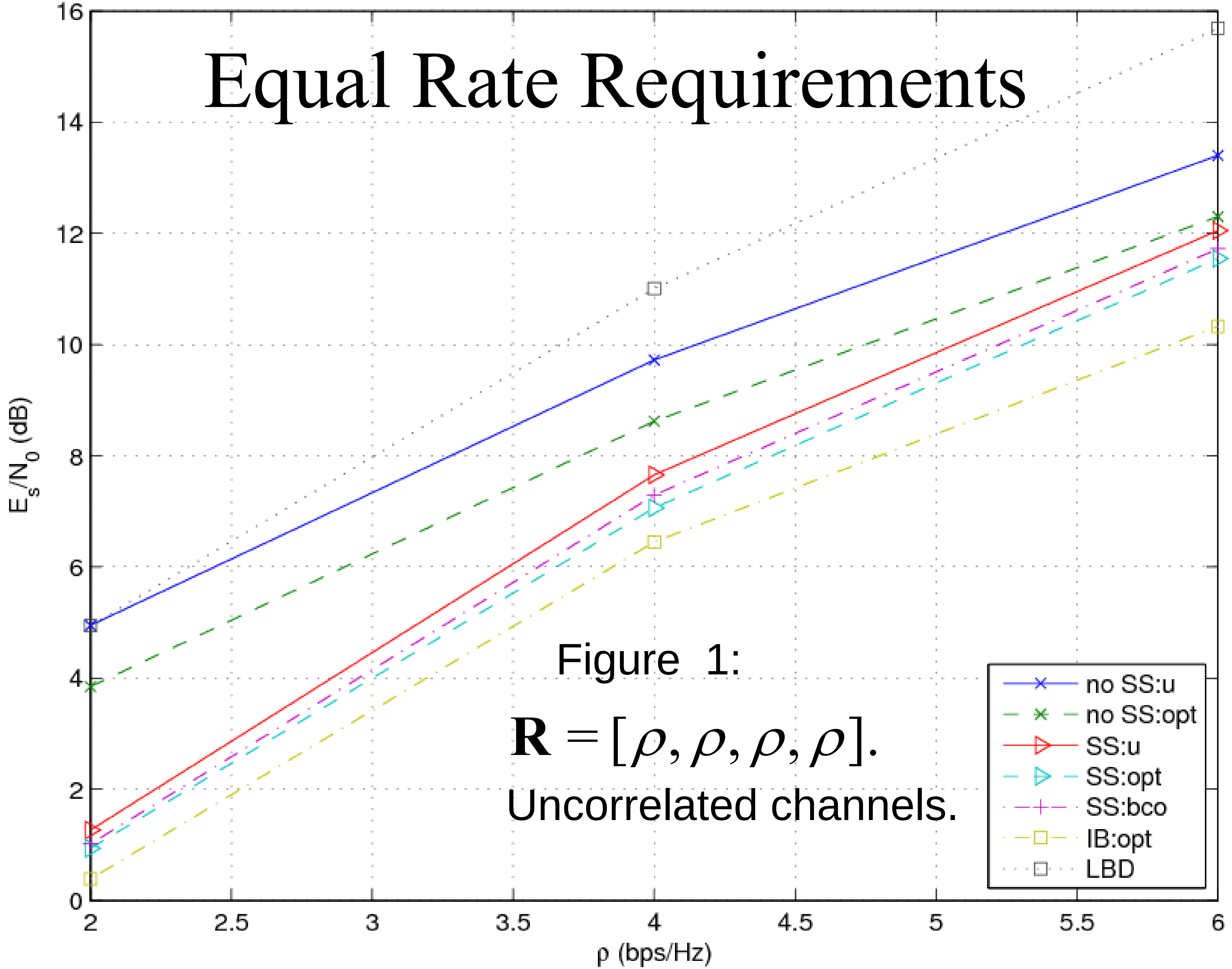



Figure 1:

$$\mathbf{R} = [\rho, \rho, \rho, \rho].$$

Uncorrelated channels.

# Equal Rate Requirements

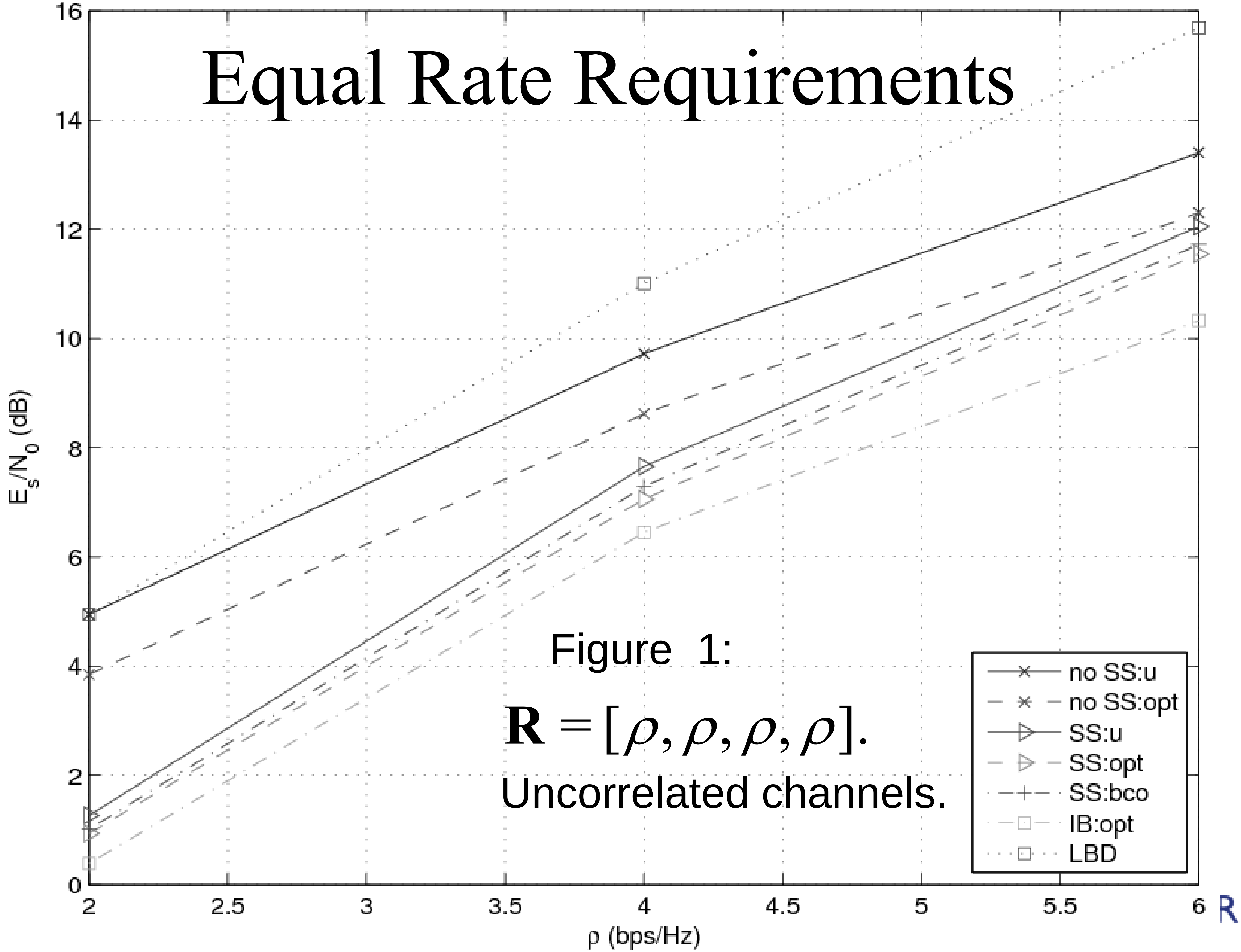



Figure 1:

$$\mathbf{R} = [\rho, \rho, \rho, \rho].$$

Uncorrelated channels.

# Unequal Rate Requirements

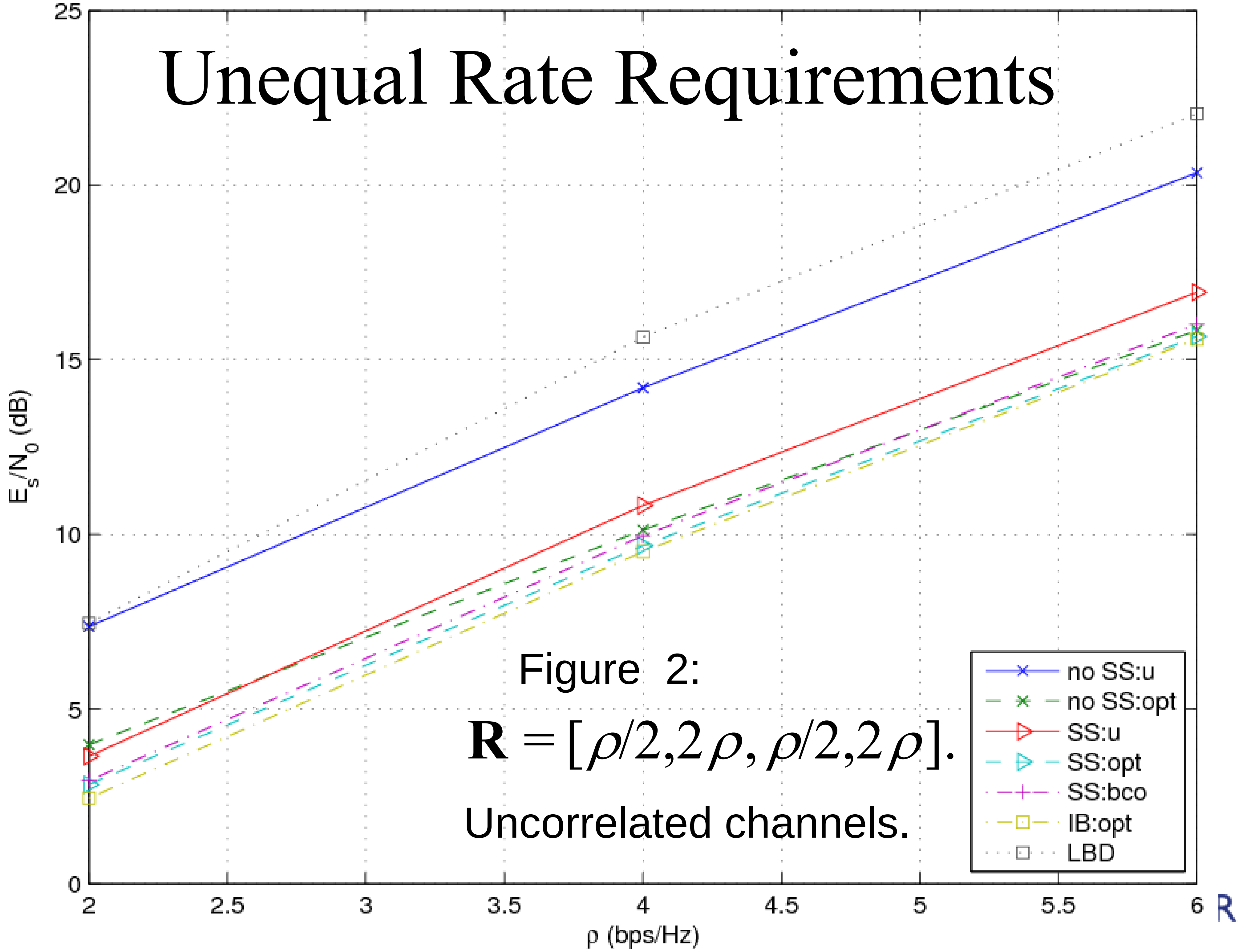



Figure 2:

$$\mathbf{R} = [\rho/2, 2\rho, \rho/2, 2\rho].$$

Uncorrelated channels.

# Unequal Rate Requirements

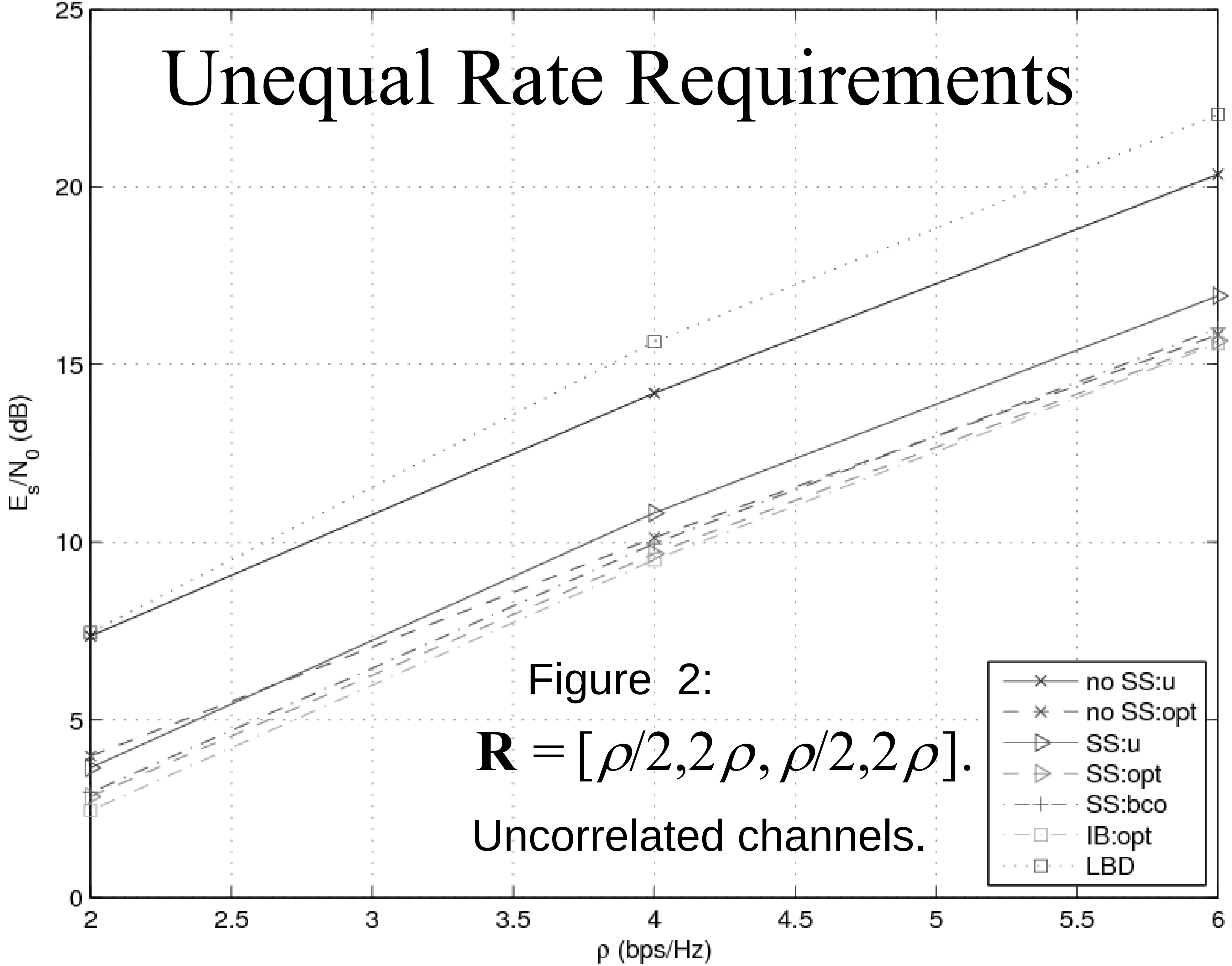



Figure 2:

$$\mathbf{R} = [\rho/2, 2\rho, \rho/2, 2\rho].$$

Uncorrelated channels.

# Correlated Channels

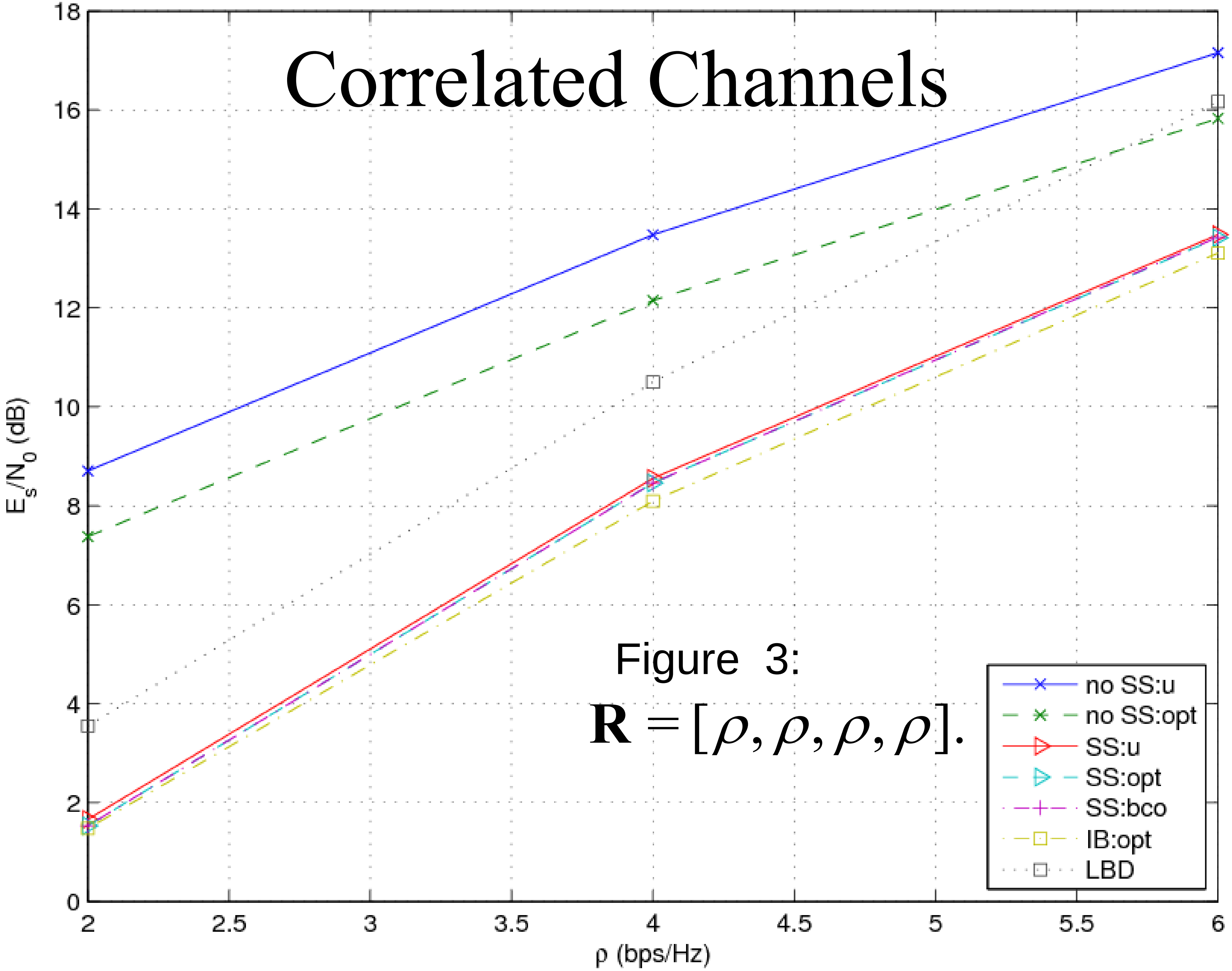



Figure 3:

$$\mathbf{R} = [\rho, \rho, \rho, \rho].$$

# Correlated Channels

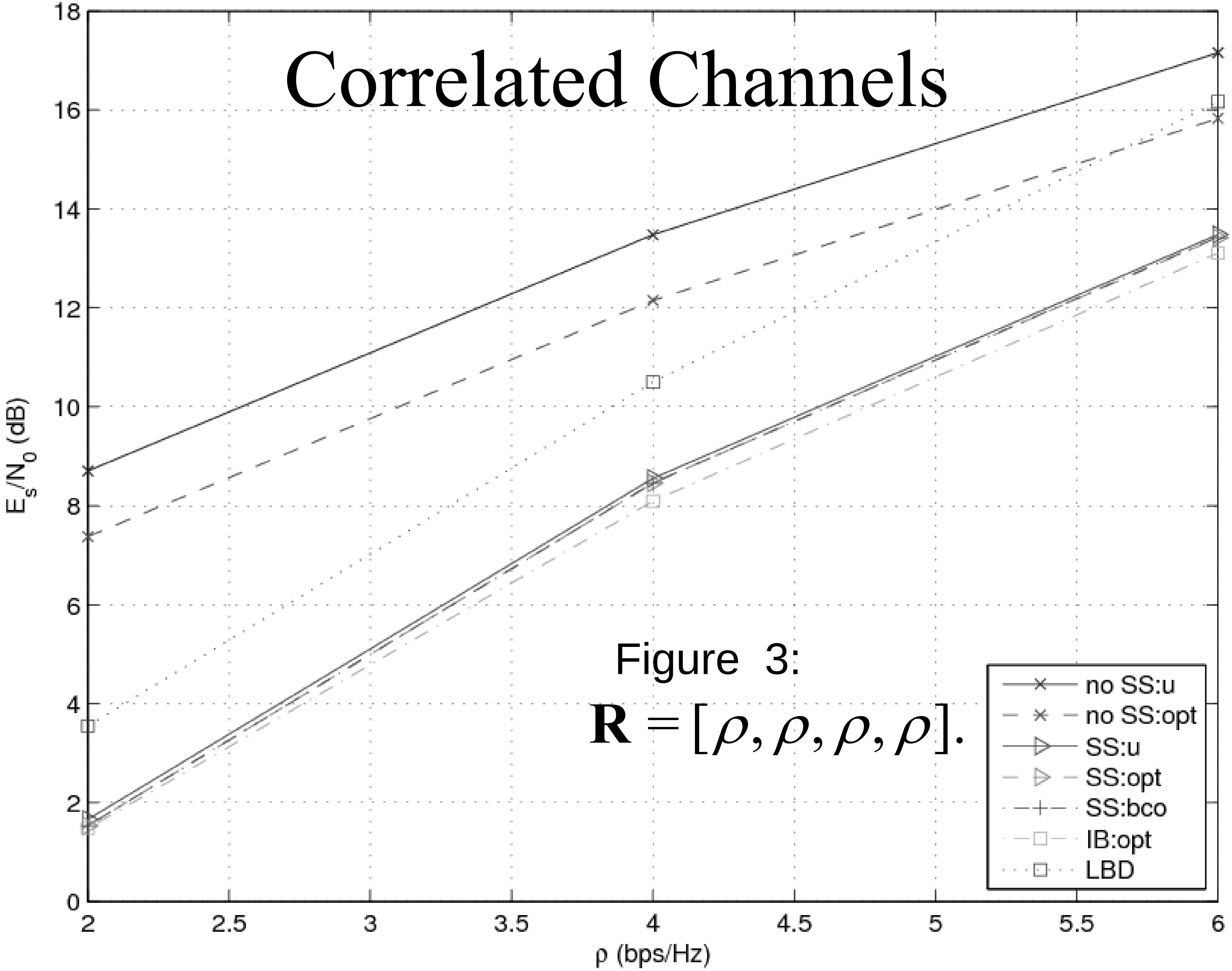



Figure 3:

$$\mathbf{R} = [\rho, \rho, \rho, \rho].$$

# Uneq. Rate, Correlated

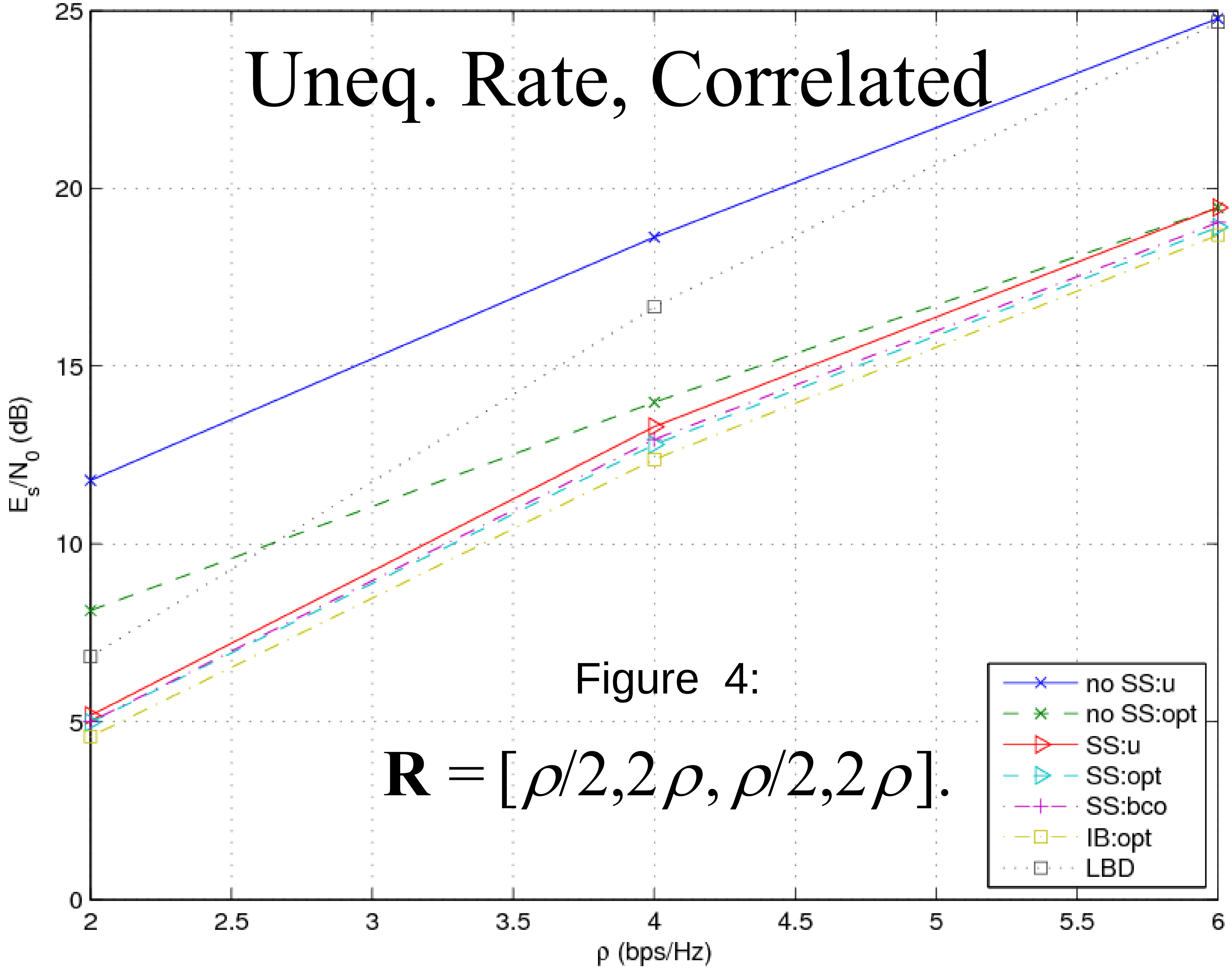



Figure 4:

$$\mathbf{R} = [\rho/2, 2\rho, \rho/2, 2\rho].$$

# Uneq. Rate, Correlated

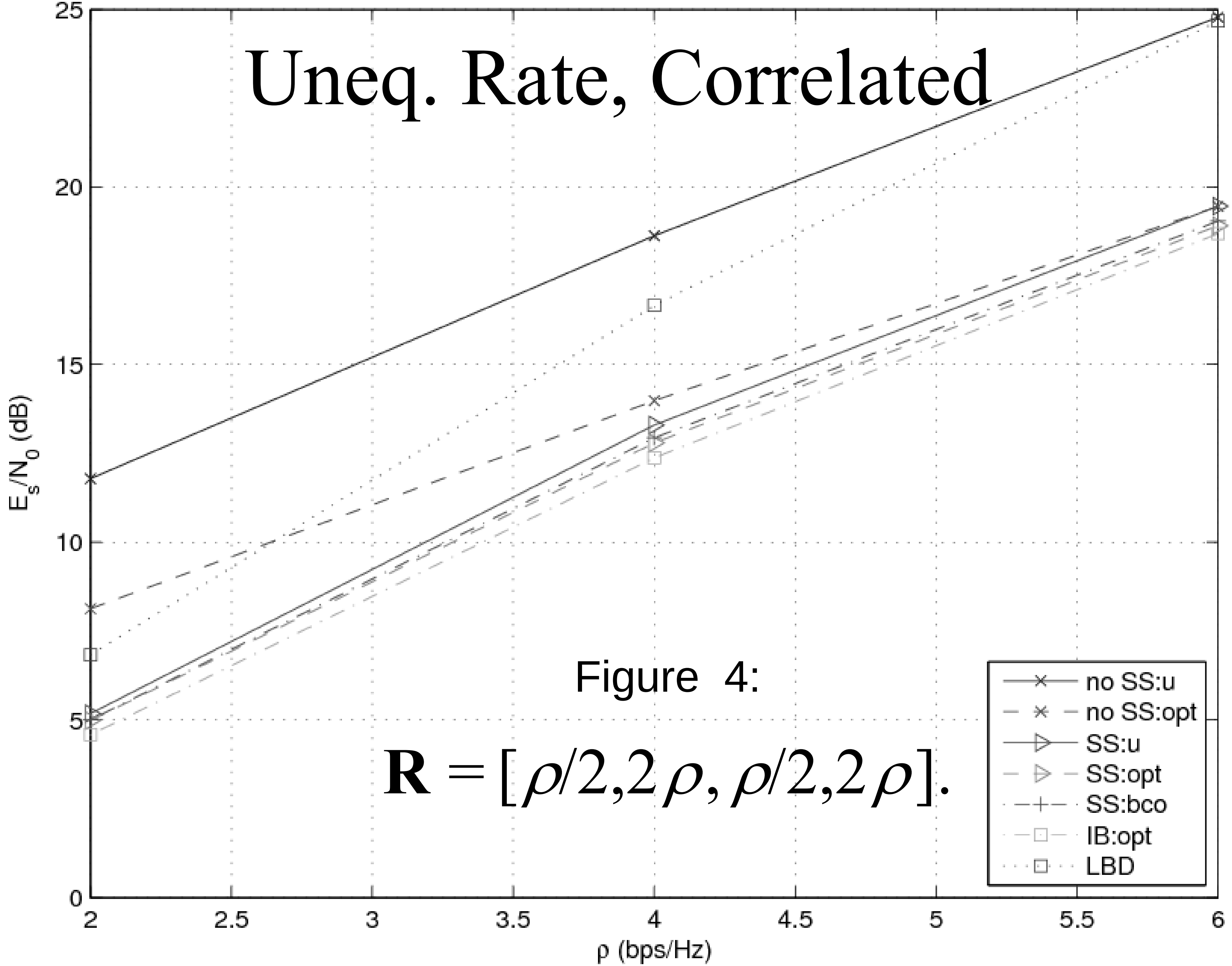



Figure 4:

$$\mathbf{R} = [\rho/2, 2\rho, \rho/2, 2\rho].$$

# Conclusion

- ZF-based Power Minimization for MIMO Broadcast Channels
- Block-diagonal Geometric Mean Decomposition (BD-GMD)
- Optimal ordering and subchannel selection
- Non-iterative solution
- Power close to IB-optimal

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